AESB 2320 Re-Exam Part 1 2016

1. a) The equation for a falling sphere is (BSLK 6.1-7)

$$f = \frac{4}{3}\frac{gD}{V_\infty^2}\left(\frac{\rho_s-\rho}{\rho}\right)$$

$Re = \frac{DV\rho}{\mu}$; ρ + μ refer to air. Therefore

$$Re = \frac{(0.007)(0.05)(1.26)}{1.75\cdot 10^{-5}} = 2.52$$

Note we can calculate Re at the start. No trial + error is needed. From Fig. 6.3-1, $f \approx 13$.

$$13 = \frac{4}{3}\frac{9.8(0.007)}{(0.005)^2}\left(\frac{\rho_s-1.26}{1.26}\right) = (3659)\left(\frac{\rho_s-1.26}{1.26}\right)$$

$$0.00355 = \frac{\rho_s-1.26}{1.26} \rightarrow \rho_s = 1.2645\ Kg/m^3$$

[One could use the Eq. on Fig 6.3-1:

$$f = \left(\sqrt{\frac{24}{Re}} + 0.5407\right)^2 = \left(\sqrt{\frac{24}{2.52}} + 0.5407\right)^2 = 13.2$$

(I guess my chart reading was close.)]

[BSL1 gives a correlation $f \approx \frac{18.5}{Re^{3/5}}$ for $1 < Re < 1000$; ~~it~~ gives $f = 10.6$]

b) The average density is (total mass)/volume

The increase in density is the result of the mass of the water film. The outer radius of the film is 0.007 m; let the inner radius be R.

$$\left(\frac{4}{3}\pi(0.007)^3 - \frac{4}{3}\pi R^3\right)1000 = \left(\frac{4}{3}\pi(0.007)^3\right)(1.2645 - 1.26)$$

Divide by $\frac{4}{3}\pi(0.007)^3$:

$$\left(1-\left(\frac{R}{0.007}\right)^3\right)1000 = 0.0045$$

$$(R/0.007)^3 = 1 - 4.5\cdot 10^{-6}$$

$$\frac{R}{0.007} = 0.9999985;\ R = 0.0069999895$$

the film is $10.5\cdot 10^{-8}$ m (10.5 nm) thick.

This is a little less than the equilibrium thickness of a soap film from colloid science. Maybe it was falling a little faster than I estimated.

Note: we didn't cover BSL1 Sect 2.6 (BSL2 sect. 2.6, BSLK sect. 2.7). The analysis there is for Re < 0.1. If you'd used that eq., then you should have checked Re and then gone to ch. 6.

2. Eq. 7.5-12 of BSLK applies

(this is the same as 7.5-10, with $D = 4R_h$)

Take surface "1" as just before inlet; $v_1 = 0$.

"2" at outlet above; $v_2 \neq 0$

What is v in pipes: $Q = 0.05 = v\frac{\pi D^2}{4} = v\frac{\pi (0.2)^2}{4} \rightarrow v = 1.59$ m/s

This is v_2 and v in all the pipes

$$Re = \frac{Dv\rho}{\mu} = \frac{(0.2)(1.59)(1.26)}{1.75 \cdot 10^{-5}} = 2.28 \cdot 10^4$$

terms in Eq. 7.5-12

- $\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}[(1.59)^2 - 0] = 1.26$
- $g(h_2 - h_1)$ - neglect *
- $\frac{1}{\rho}(p_2 - p_1) = \frac{1}{1000} \cdot 0 = 0$
- $2v^2 \frac{L}{D} f(Re)$: what is f? $\frac{K}{D} = \frac{0.0005}{0.2} = 0.0025$; $Re = 2.28 \cdot 10^4$

  from Fig 6.2-2 (BSLK), $f \cong 0.0075$

  $2(1.59)^2 \frac{(2+1+1+2)}{0.2}(0.0075) = 1.14$

- rounded entrance: $e_v = 0.05$ BSLK Table 7.5-1

  2 × rounded elbow: $\sim 0.65 \times 2 = 1.3$ "

$$\Sigma \tfrac{1}{2} v^2 e_v = \tfrac{1}{2}(1.59)^2(1.3 + 0.05) = 1.71$$

$\rightarrow 1.26 + 0 + 0 = \hat{W}_m - 1.14 - 1.71 \rightarrow \hat{W}_m = 4.11$

this is work / unit mass. To convert to work / time

$W_m = w\hat{W}_m = (Q\rho)\hat{W}_m = (0.05)(1.26)\,4.11 = 0.26$ Watts

This problem was inspired by the hood pipes one can see in the roof of the Steamlab. (I changed the lengths of the straight sections of pipe.)

* The reason one can neglect gravity is as follows: the system pumps air upwards in a sea of air (the atmosphere). The small term $g(h_2 - h_1)$ would be exactly balanced by the slight difference in pressure of surrounding air over a change of height of 4 m. The two differences cancel

3. a) For laminar flow in a slit, $Q = \frac{2}{3} \frac{\Delta p \, B^3 W}{\mu L}$

$$6 \cdot 10^{-7} = \frac{2}{3} \frac{2 \cdot 10^4 B^3 (0.05)}{(0.001)(0.3)} = 2.22 \cdot 10^6 B^3$$

$B = 6.46 \cdot 10^{-5}$ m $\quad 2B = 1.29 \cdot 10^{-4}$ (129 μm)

[This flow rate would correspond to a permeability of about 5 Darcy if it were flow through rock. A narrow fracture gives as much flow as a rock of high permeability.]

b) For a slit, $Re = \frac{4BV\rho}{\mu} = \frac{2.58 \cdot 10^{-4} V \, 1000}{0.001}$

What is v? $V = \frac{Q}{A} = \frac{6 \cdot 10^{-7}}{1.29 \cdot 10^{-4} \times 0.05} = 0.093$

$Re = 258(0.093) = 23.99$

Flow is turbulent if $Re > 2300$. Not so.

Assumption of laminar flow is valid.

[see note below]

4. This problem differs from BSL1 sect. 2.3 in

BC at $x = 0$

" $\quad x = \delta$

type of fluid

The first of these to come up in the derivation is the B.C. at $x=0$ (Eq. 2.2-10). Thus the last eq. that applies is Eq. 2.2-9

Note: some students solved using the hydraulic radius approx. That is valid only for turbulent flow. This case is laminar. Note also that in the hydraulic radius approx, $V = \frac{Q}{A}$ where A = true area of conduit (slit here), not area of a tube.